

Stringy Model of Cosmological Dark Energy

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Abstract. A string field theory (SFT) nonlocal model of the cosmological dark energy providing $w < -1$ is briefly surveyed. We summarize recent developments and open problems, as well as point out some theoretical issues related with others applications of the SFT nonlocal models in cosmology, in particular, in inflation and cosmological singularity.

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INTRODUCTION

The origin of the dark energy (DE) is still a fascinating puzzle. Present cosmological observations do not exclude an evolving DE state parameter w . According recent data (U.Seljak, Pascos, July 07)

$$w = -1.04 \pm 0.06.$$

There are two questions to experimental data:

- Can we rule out a dynamical DE?
- Can we rule out $w < -1$?

Recent data are not enough to answer these two questions and moreover, within the next few years answers on these two questions will not accessible as it has previously expected. However one might wonder whether there is a room for $w < -1$ in a theory.

Dark energy models with the state parameter $w < -1$ violate the null energy condition (NEC). All local models realizing the NEC violation are unstable and violate usual physical requirements. To provide $w < -1$ a string field theory (SFT) nonlocal DE model has been proposed [1].

In this SFT nonlocal model our Universe is considered as a D3 non-BPS brane embedded in the 10 dimensional space-time. The role of the dark energy plays the Neveu-Schwarz (NS) string tachyon leaving in GSO- sector. The tachyon action is dictated by the cubic fermionic SFT [2, 3] and it is nonlocal due to string effects [4].

We postulate a minimal form of the tachyon interaction with gravity. In the spatially flat FRW metric the model is described by a system of two nonlinear nonlocal equations for the tachyon field and the Hubble parameter. The corresponding potential has perturbative and nonperturbative minima. A transition from a perturbative vacuum to a non-perturbative one is interpreted as D-brane decay. It happens that this model under

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some conditions displays a phantom behaviour [5]. Note that unlike phenomenological phantom models here phantom appears in an effective theory. Since SFT is a consistent theory this approach does not suffer from usual problems which are inevitable for phenomenological phantom models. The UV completion is supposed to be solved by extending the one mode (tachyon) approximation.

SFT in the flat background dictates a particular value of the D-brane tension. It can be found from the requirement that the total energy of the system in the true non-perturbative vacuum is zero. In cosmology this total energy of the system in the true non-perturbative vacuum can be interpreted as the cosmological constant. It has been conjectured that an existence of a rolling solution describing a smooth transition to the true vacuum does define the value of the cosmological constant [1]. We cannot prove this conjecture but arguments to it favor can be given using the local approximation [8]. A recent breakthrough in solving numerically the full nonlinear and nonlocal system of equations [7] also supports this expectation.

MODEL

Our model is given by the following action [1]

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{2\kappa^2} + \frac{1}{\lambda_4^2} \left(-\frac{\xi^2 \alpha'}{2} g^{\mu\nu} \partial_\mu \phi(x) \partial_\nu \phi(x) + \frac{1}{2} \phi^2(x) - \frac{1}{4} \Phi^4(x) - T' \right) \right)$$

Here $g_{\mu\nu}$, κ and λ_4 are the four-dimensional metric, gravitational coupling constant and scalar field coupling constant, respectively; $\frac{1}{\lambda_4^2} = \frac{v_6 M_s^4}{g_o} \left(\frac{M_s}{M_c} \right)^6$, g_o is the open string dimensionless coupling constant, M_s is the string scale $M_s = 1/\sqrt{\alpha'}$ and M_c is a scale of the compactification, v_6 is a number related with a volume of the 6-dimensional compact space. $T' = 1/4 + \Lambda'$, where Λ' is a dimensionless cosmological constant. ϕ is a tachyon field and Φ is related with ϕ by the following relation $\Phi = e^{\frac{\alpha'}{8} \square_g \phi}$, where $\square_g = \frac{1}{\sqrt{-g}} \partial_\mu \sqrt{-g} g^{\mu\nu} \partial_\nu$. This form of the nonlocal interaction is defined by the cubic fermionic SFT (CFSFT) [3]. More precisely, the CFSFT brings a more complicated form of the interaction, but by an analogy with the flat case [6, 5, 7] we believe that this approximation catches essential physical properties of the model and $\xi^2 \approx 0.9556$ is a constant defined by the CFSFT.

In the spatially flat FRW metric with a scale factor $a(t)$ the equation for a space homogenous tachyon field Φ and the Friedmann equations have the form [1]

$$(\xi^2 \mathcal{D} + 1) e^{-\frac{1}{4} \mathcal{D}} \Phi = \Phi^3, \quad \mathcal{D} = -\partial_t^2 - 3H(t) \partial_t, \quad H = \partial_t a / a \quad (1)$$

$$3H^2 = \frac{\kappa^2}{\lambda_4^2} \left(\frac{\xi^2}{2} \partial_t \phi^2 - \frac{1}{2} \phi^2 + \frac{1}{4} \Phi^4 + \mathcal{E}_1 + \mathcal{E}_2 + T' \right), \quad (2)$$

$$\mathcal{E}_1 = -\frac{1}{8} \int_0^1 ds \left((\xi^2 \mathcal{D} + 1) e^{\frac{s-2}{8} \mathcal{D}} \Phi \right) \cdot \left(\mathcal{D} e^{-\frac{1}{8} s \mathcal{D}} \Phi \right), \quad (3)$$

$$\mathcal{E}_2 = -\frac{1}{8} \int_0^1 ds \left(\partial_t (\xi^2 \mathcal{D} + 1) e^{\frac{s-2}{8} \mathcal{D}} \Phi \right) \cdot \left(\partial_t e^{-\frac{1}{8} s \mathcal{D}} \Phi \right). \quad (4)$$

The non-local energy \mathcal{E}_1 plays the role of an extra potential term and \mathcal{E}_2 the role of the kinetic term. Note that here we use a dimensionless time $t \rightarrow t\sqrt{\alpha'}$.

HOW WE STUDY OUR MODEL AND WHAT WE GET

Equations (1) and (2) form a rather complicated system of nonlinear nonlocal equations for functions Φ and $H(t)$ because of the presence of an infinite number of derivatives and a non-flat metric. Before to discuss the methods of study this model let us mention the known methods of study equation (1) in the flat background, $H = 0$,

$$(-\xi^2 \partial_t^2 + 1) e^{\frac{1}{4}\partial_t^2} \Phi(t) = \Phi(t)^3. \quad (5)$$

Equation (2) in the flat case describes the energy conservation [5].

A boundary problem $\Phi(\pm\infty) = \pm 1$ for (5) has been studied using:

- a numerical method [6] based on an integral representation of (5); it is related with a diffusion equation method [9] which uses an auxiliary function of two variables $\Psi(r, t)$ that is the subject of a linear equation and $\Psi(\frac{1}{4}, t) = \Phi(t)$;
- a decomposition on local fields [10, 11, 12]; this method works well for linear equations and has been used to study solutions to (5) near vacuum ± 1 ;
- existence theorems [13, 14, 15];
- almost exact solutions methods [16, 17]; the approach [16] uses a diffusion equation method.

The following two characteristic properties of (5) have been obtained

- an existence of a critical point $\xi_{\text{cr}}^2 \approx 1.38$ such that for $\xi^2 < \xi_{\text{cr}}^2$ eq. (5) has a rolling solution [6] interpolating between ± 1 ;
- an existence of a dominance of an extra non-local kinetic term \mathcal{E}_2 over the local kinetic one [5] and as a result, an appearance of a phantom behavior providing $w < -1$.

These result have been obtained using numerical calculations. It is very interesting to study the problem analytically and also try to find approximate models admitting explicit solutions and having above mentioned properties. They could be two or more components local models.

An investigation of non-flat eqs. (1) and (2) is essentially more complicated. The following methods are used:

- A decomposition on local fields and a modification of the potential have been used in [8, 17, 19, 12]. A simplest one phantom mode approximation with an explicit form of the solution $\phi(t) = \tanh(t)$ is realized for a six-order potential [8] and gives $H_0 = 1/3m_p^2$. Assuming that $M_c \sim M_p$ and $M_s \sim 10^{-6.6}M_p$ we get

$$H_0 \sim 10^{-60}M_p.$$

- an analytic approach that is closely related with the diffusion equation method [20].

- A numerical study has been performed in [7], where the diffusion equation method has been used to define $\exp \mathcal{D}$ and a double-step iteration procedure has been proposed.

The following physical effects are found in [7]

- For $\xi^2 < \xi_{\text{cr}}^2 \approx 1.18$ and $\Lambda = \Lambda(\xi)$ the system (1), (2) has a rolling solution.
- For $\xi^2 < \xi_{\text{shape}}^2$ and $t > 0$ the Hubble function $H(t)$ is a function which has small fluctuations about a monotonic function $H_l(t)$ with an asymptotic H_0 ; for $\xi_{\text{shape}}^2 < \xi^2 < \xi_{\text{cr}}^2$ $H(t)$ describes fluctuations about a function $H_l(t)$ that has two maximum. To realize this approximated shape $H_l(t)$ by local fields one needs at least two fields [19].

As in the flat case it would be very interesting to find approximate analytical solutions which exhibit these properties. Note that two maximum shape regime for $H(t)$ is interesting in a context of building an unified cosmological evolution. Let us also note that there are applications of non-local SFT models to inflation [21, 22] and cosmological singularity (see refs. in [12]).

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